# Hawkes Process, Risk Modeling & Applications

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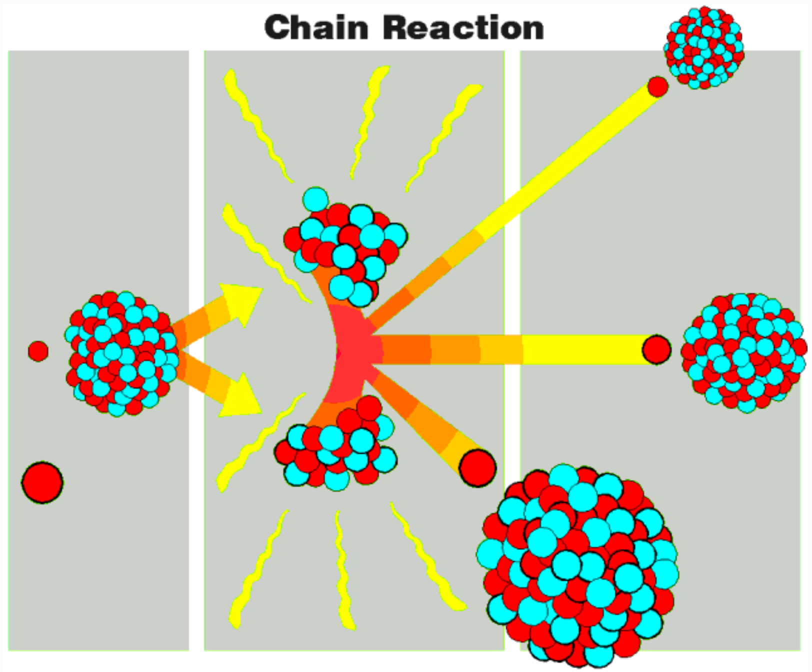
Insurance Market Conferences

# Outline

- Background
- What is Hawkes Process?
- Why new modelling with HP?
- Modelling Cyber Risk & Information Security
- Application for Social Media
- Predicting earthquakes & natural catastrophe
- Predicting crimes and terrorist attacks
- Tracking banana trees
- Finance
  - Insurance
  - Credit Risk
  - Mini-Flash Crash
- Q & A

# Background

There are many events occurring in sequence: one event often triggers a series of similar events in a limited space of time. E.g. earthquake aftershock sequences or arrivals of claims. A good way to think of this is to consider Domino effects or chain effects.



# Background

One feature is that similar type of events become more likely to happen. Mathematically, we say that the occurrence of certain events may increase the probability of future events occurring soon after.

These features are very important. One example which we may easily relate to is contagion effects occurred during the recent financial crisis. This, understandably, has made regulators, practitioners, economists and almost everyone concerned that risk, cost of risk and financial losses could be too high for a long period.


# What is Hawkes Process

In the 1970s, Professor Alan Hawkes introduced a family of probability models for the occurrence of series of events.

He developed both self-exciting and mutually-exciting processes. These later came to be called Hawkes processes.

They began to be used in seismology - known as ETAS models (Epidemic Type Aftershock Sequences).

# Why New Modelling with HP?

Classic financial models are based on a smooth evolutionary process such as Brownian Motion or Poisson processes. Such models usually use the latest available observation for forecasting. The best forecast of tomorrow's price will be based on today's price plus some kind of change within certain error bounds driven by some information shocks (such as news events). But the history of the evolution of risk or price movements have no inputs. This doesn't sound quite right.

Often, risk or price processes of an asset will follow a random walk path with normal distribution, which is novel and beautiful. However, we all know the real financial data tend to be non-normal and have skewed distributions or fat/heavy/long & thin tails.

No contagion can be captured by classic models. For example, When Thailand flood occurred, we know the claims also flooded in and underwriters got a lot busier. No consideration of such chain effects would further affect the calculation of probability of ruin and subsequently, for example, the adjustment of future premium or a company's reserve level.

# HP Fact Sheet

- ✓ Provide probability of occurrence of an event by incorporating the entire history (conditional probability); therefore, more accurate information will be fed into further account of things such as capital allocation, cash level prediction etc.;
- ✓ Great capacity of incorporating underlying distributions, including those fit for extreme events;
- ✓ Capture contagion and clustering effects;
- ✓ Easy to extend to real continuous-time model.

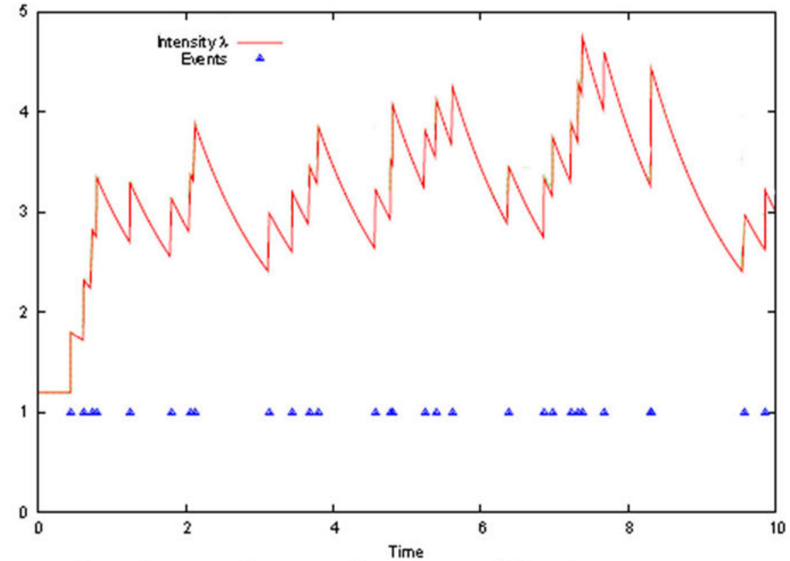
# What have they been applied to?

Over the last 15 years their use has spread to many subjects.

- Ecology (Whale spotting, spider colonies, invasive banana trees)
- Crime prediction (gang fights in LA, burglary and car theft in LA, Santa Cruz and Kent (UK))
- Terrorist acts (Iraq, Indonesia)
- Molecular genetics; neuroscience; recurrence of cancer tumors
- Social networks (Twitter, Facebook, YouTube, conversation analysis)
- Finance (HFT, Insurance, Credit Risk, Risk Management)

# Self-exciting HP

$$\mathrm{P}(t) = \text{Continuous evolution} + \text{Self-exciting Immigration}$$


Simulation of a one-dimensional Hawkes process

# Mutually-exciting HP

Let's assume there are n types of events occurring in the market, the intensity of type-i events is not only raised by the lagged events of the same type but also by different type events (e.g. type-j) events.

$$P\left(t\right) = \text{Continuous Evolution} \\ + \left(\text{Self-excitation}\right)\left(\text{Cross-excitation}\right)$$

For example, price changes of Apple stocks on NYSE may be followed by price changes of Apple on LSE.

# Marked HP

We also can assign the value associated with the type-j event. *e.g.* the magnitude of an earthquake; the size of a jump in the price of a stock on a financial market, then a large jump may increase the probability more than a small jump.

$$P(t) = \text{Continuous Evolution}$$
$$+ [(\text{Self-excitation})(\text{Cross-excitation}), Z_{c,t}]$$

# Cyber Risk

Businesses Aren't Alone in Worrying About Cyber Risk
Wednesday, 14 October 2015 - *Insurance Journal*

Asset Managers Urged to Make Cyber Risk Top Priority
Thursday, 3 September 2015 - *Insurance Journal*

Marsh Partners with FireEye on Cyber Risk Management Program
Tuesday, 19 May 2015 - *Insurance Journal*

Marsh survey finds more focus on cyber risks
Tuesday, 28 April 2015 - *Business Insurance*

Congress seeks solutions on cyber risk that include insurance
Monday, 30 March 2015 - *Business Insurance*

U.K. government, insurers double down on cyber risk
Tuesday, 24 March 2015 - *Business Insurance*

Cyber risk models take cues from natural catastrophe tools
Monday, 23 March 2015 - *Business Insurance*

Willis Expert: Aviation Industry is Under-Prepared to Deal with Cyber Risks
Monday, 9 March 2015 - *Insurance Journal*

Cyber risks a top concern for majority of D&O underwriters
Tuesday, 3 March 2015 - *Business Insurance*

Catastrophe modelers developing cyber risk technologies to assess exposures
Monday, 5 January 2015 - *Business Insurance*

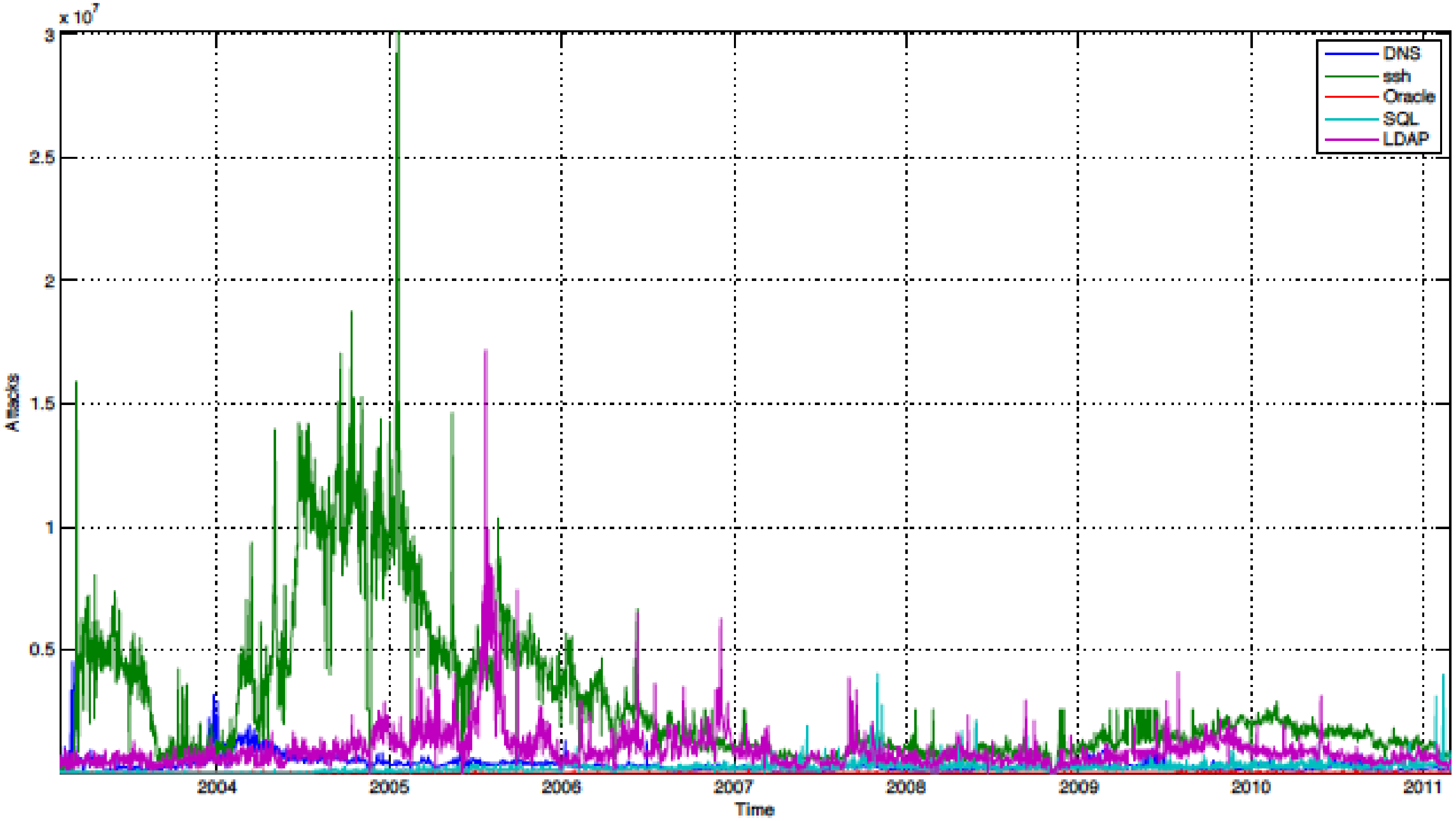
U.S. urges banks to consider cyber risk insurance amid hacking threats
Thursday, 4 December 2014 - *Business Insurance*

Half of Irish firms unprepared for cyber risks: EY
Monday, 1 December 2014 - *Business Insurance*

Interview: D&O Cyber Liability, Cyber Risks Top Lawyers' Agenda
Thursday, 2 October 2014 - *Insurance Journal*

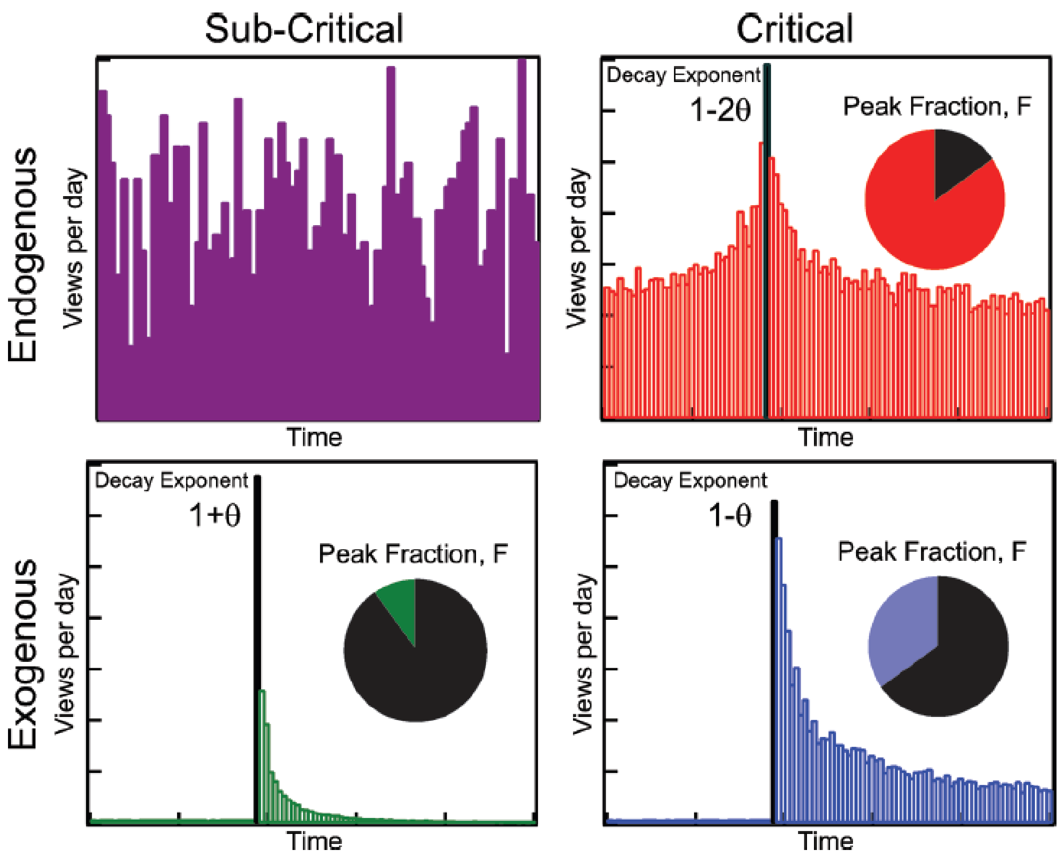
Credit card council releases cyber risk mitigation guidance
Tuesday, 12 August 2014 - *Business Insurance*

# Cyber security Attack



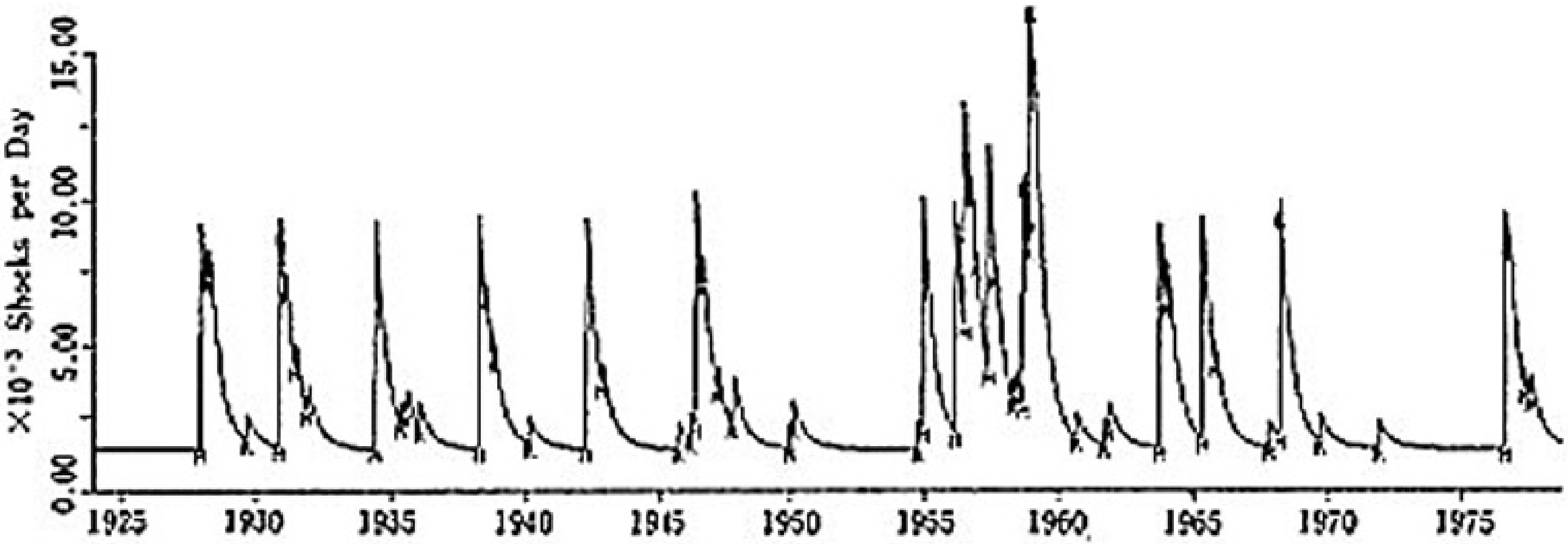

Baldwin et al (2012) suggest that Hawkes processes provides better modelling cyber attacks into information systems because it captures jumps shocks and persistent after shocks that may form attack contagion.

# Social Media




Crane & Sornette (2008) suggest that classic Poisson process do not accurately describe clustering effects after studying both endogenous and exogenous bursts of activities of watching YouTube clips. In contrast, HP modelling gives much detection of epidemic cascade of such actions.

# Earthquakes



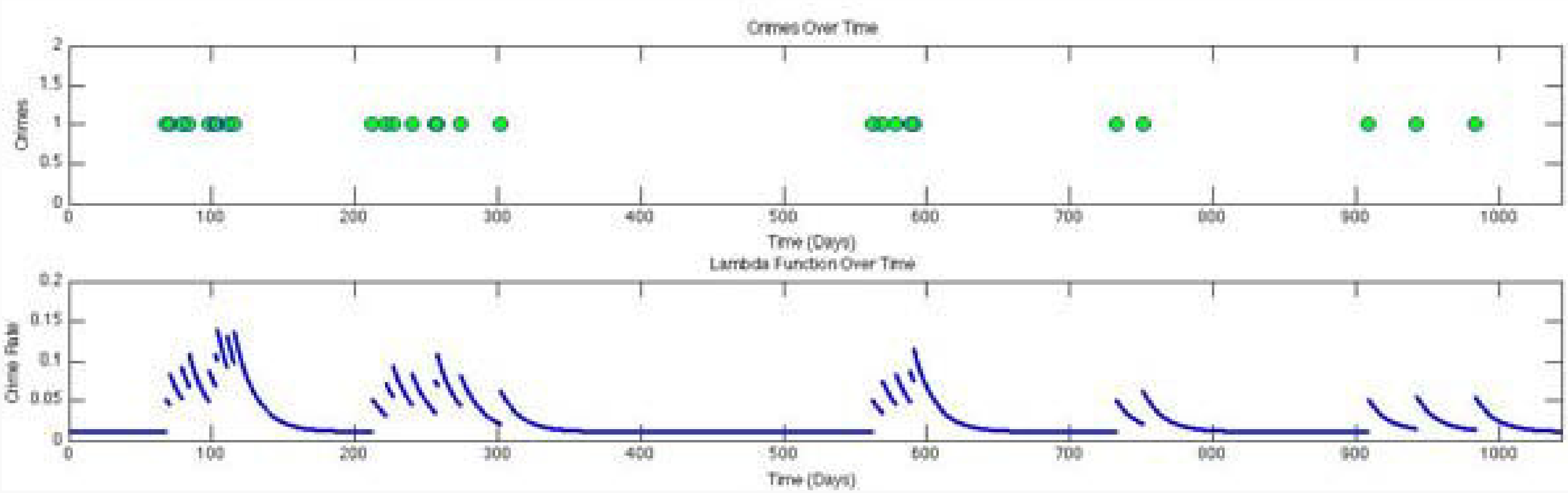

**Estimated intensity process of Kwanto earthquakes**

**The symbols K and H indicate the occurrence times of earthquakes in Kwanto and Hida area, respectively.**

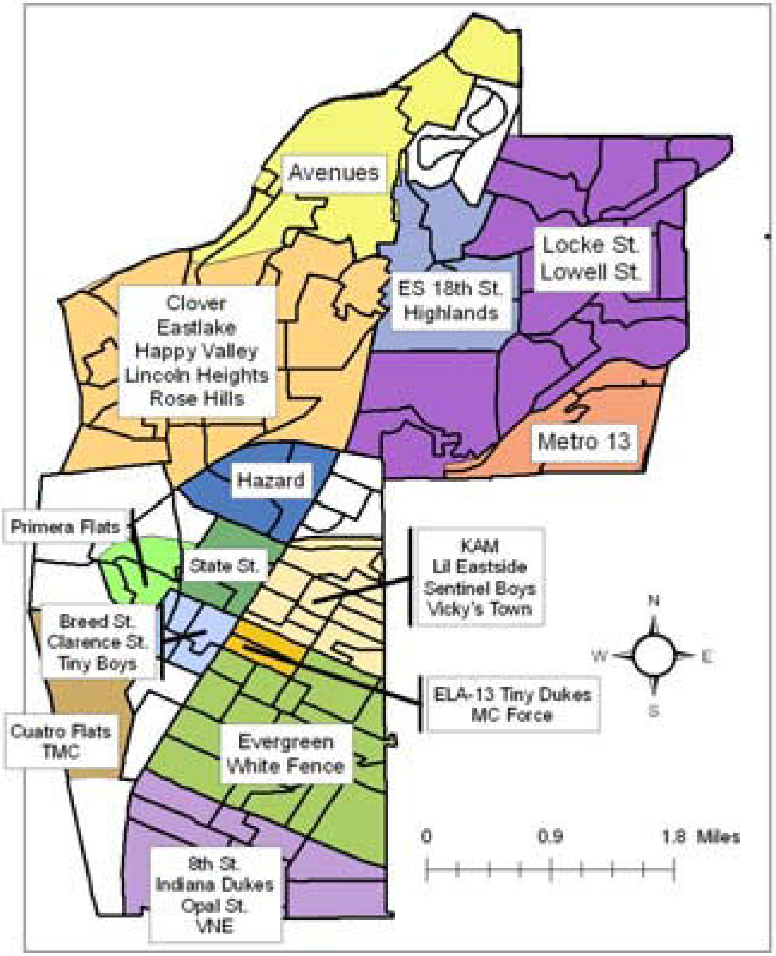
Ogata et al (1982) find that K area often has large earthquakes but H area has small ones. But the large earthquakes in K are more triggered by frequent small quakes in H.

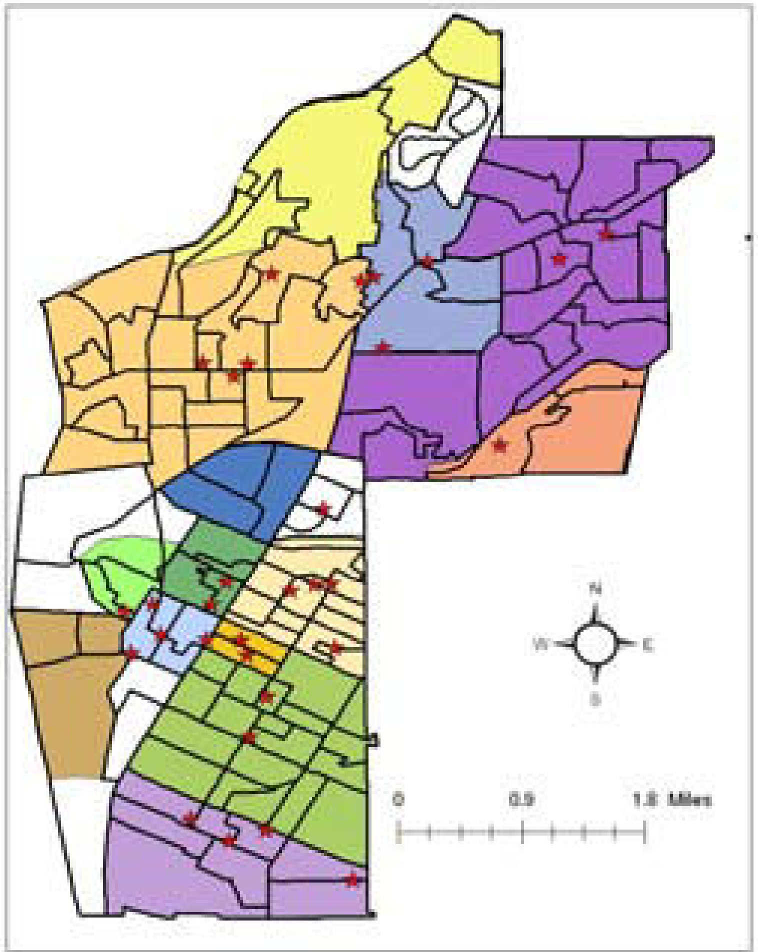
# Crime: Gang warfare in LA

Applications to crime have been published by Egesdal, Fathauer, Louie & Neuman (2012) in Los Angeles. Rivalry between two gangs can be modelled by considering two kinds of events: gang *a* attacks gang *b*, and gang *b* attacks gang *a*. Following an event of one type the next event might be another similar attack or a retaliation by the other gang.



# Crime: Gang warfare in LA



# Crime: Burglary & Grand Theft Auto

## Sending the Police Before There's a Crime

Jim Wilson/The New York Times

Officers at a location flagged by a computer program as a place where car burglaries were especially likely put a woman in custody.

By ERICA GOODE

Published: August 15, 2011

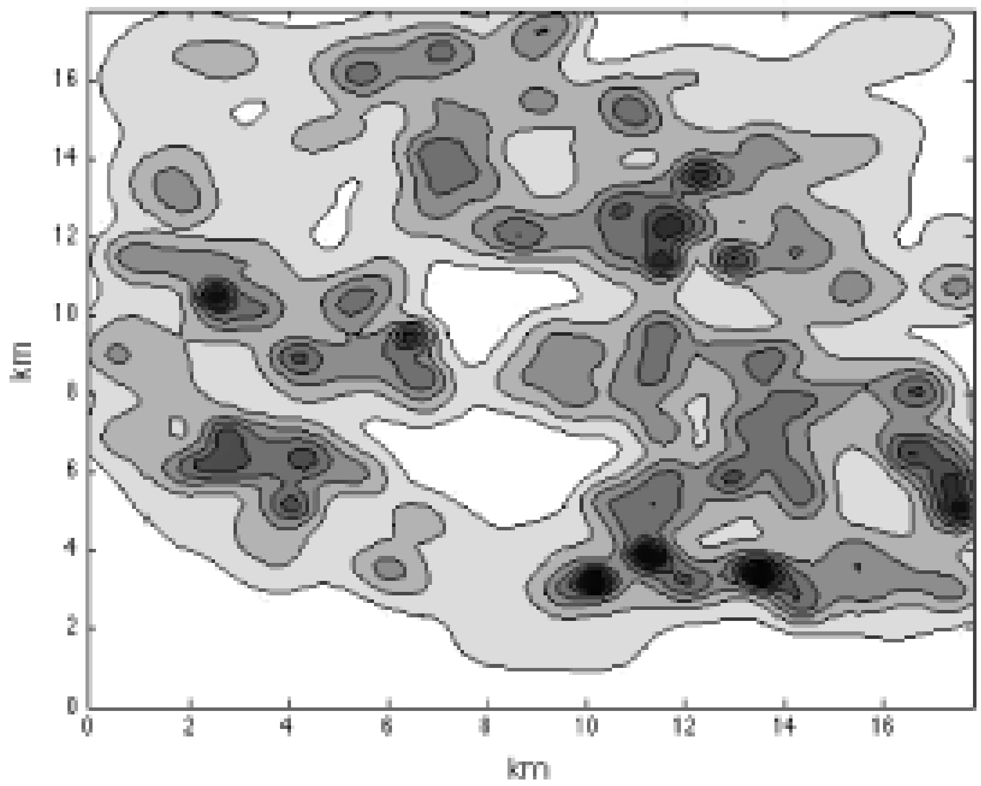
*From the New York Times*

http://www.nytimes.com/2011/08/16/us/16police.html?module=Search&mabReward=relbias%3Aw

# Crime: Burglary & Grand Theft Auto

We are aware police in Santa Cruz and the Los Angeles Police Department have been using computer software based on Hawkes Processes to anticipate where and when crimes were most likely to occur, turn up and make an arrest. Although the title of ‘Hawkes processes’ was not directly quoted in their documents or news release, George Mohler, an academic researcher in the University of California in building these models, confirmed that the program was based on Hawkes processes and he provided the information that Kent (UK) Police also started using it in December 2012.

# Crime: Burglary & Grand Theft Auto




Using a marked HP, we can provide the location measure indicating the potential heavy vs. light crime areas (dark vs. light spots). The police, then, can allocate their force and take appropriate actions for prevention or tackling crimes.

# Biology: Invasive Banana Tree

E. Balderama (2012) studied the spread of red bananas, musa-velutina, in Costa Rica using the same type of spatio-temporal model as the Hawkes earthquake models. They found about 58% of new banana trees are triggered by existing ones in the region. The remaining plants are immigrants from outside.

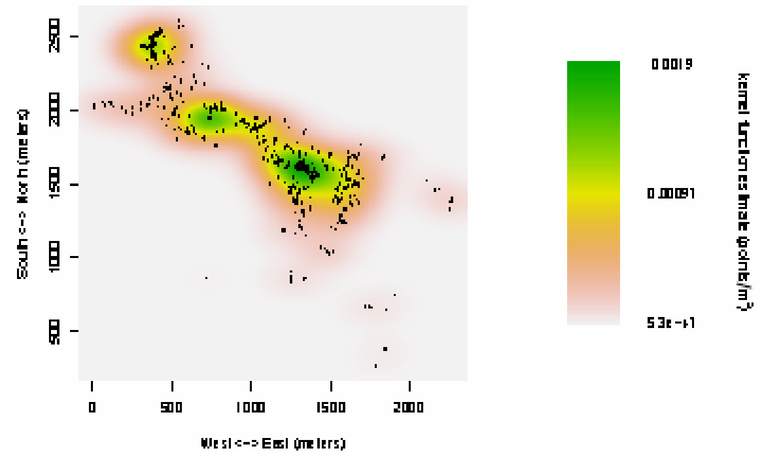



Figure 6.2: Estimated background rate $\hat{\mu}(x, y)$ obtained by kernel smoothing, along with all observed plants (black points).

# Insurance

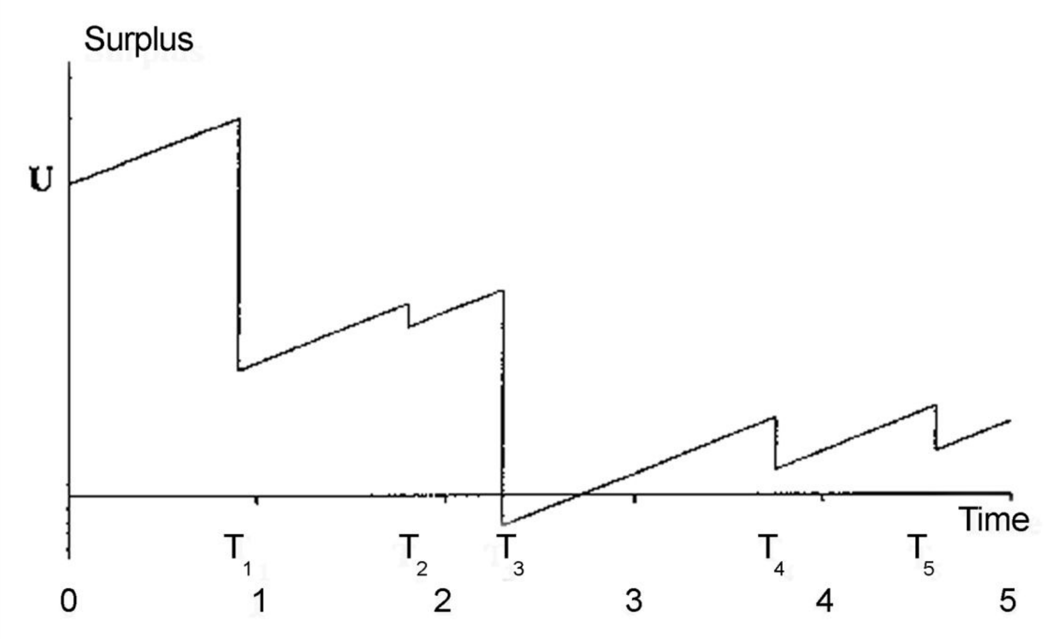
The surplus process $U(t)$ is written as:

$$U(t) = U + ct - \sum_{i=1}^{N(t)} X_i$$

*U: the initial capital;*

*c: the rate at which premium income is paid; claims are paid out at times that occur as a continuous (Poisson) process;*

*Claim sizes $\{X_i\}$ are i.i.d.*



# Insurance

Traditionally, the arrival of claims has been modeled by a Poisson process. Stabile and Torrisi (2010) replaced this by a simple Hawkes process in studying the classical problem of the probability of ruin.

Dassios and Zhao (2011) considered the same ruin problem using a more complicated marked mutually-exciting process (dynamic contagion process). Jang and Dassios (2012) implement Dassios & Zhao (2011) to calculate insurance premiums and suggest *higher premiums* should be set up in general across different insurance product lines. It is because, in comparison to simple Poisson arrival of claims, these models also account for clustering of claims.

# Credit Risk

Let us suppose you have a portfolio composed of $n$ companies and are interested in understanding the potential defaults (company goes bust) of this portfolio. One reason for modelling this process is to set prices for collateral debt obligations (CDOs), which are, by design, insurance that protects against loss in these circumstances.

# Credit Risk

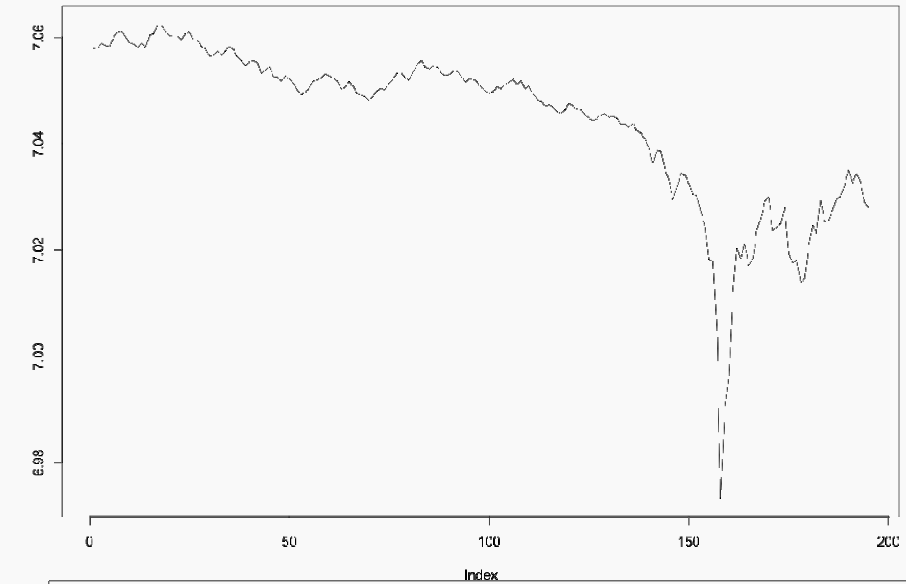
Lando studies $n$ = 2557 companies over 1982-2005, having a total of 370 defaults. For $i$th indivdual company the intensity is given by

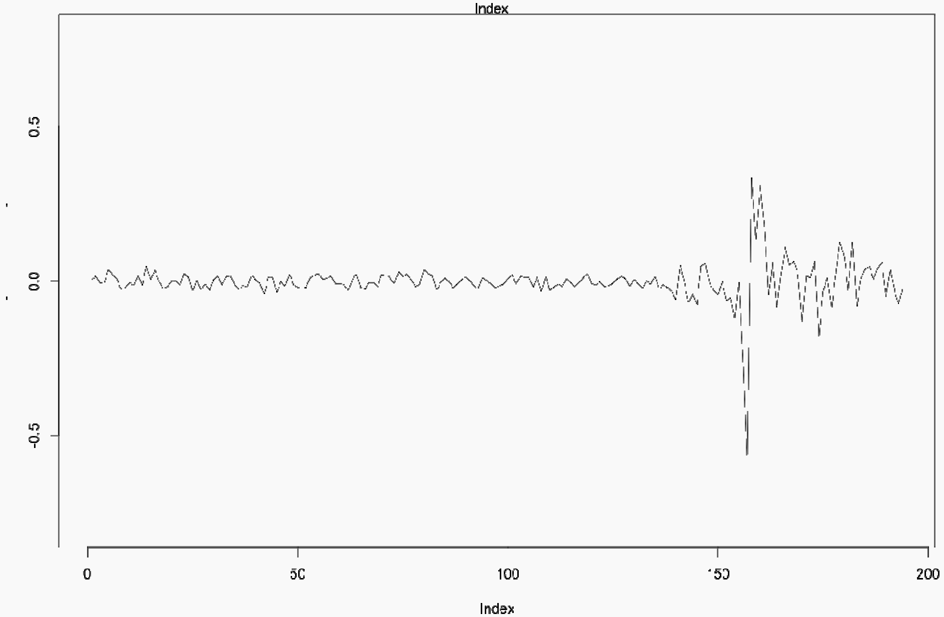
$$\lambda_i(t) = R_{i,t}\left(\delta + f\left(macro\right) + f\left(FirmFundamentals\right) + f_{HP}\left(N_s, Y_s\right)\right)$$

- *f(macro)* contains4 macroeconomic variables
- *f(FirmFundamentals)* contains 5 company-specific variables
- $N_s$ is the point process of defaults in the portfolio
- $Y_s$ is the size (book asset value) of the company that defaults at time $s$
- $R_{i,t}$ equals 1 if company at risk at time $t$; equals 0 otherwise

With an assumed distribution of loss sizes, the model can forecast the future losses and calculate the rate of premium payments, hedging and credit derivatives.

# Mini-Flash Crash



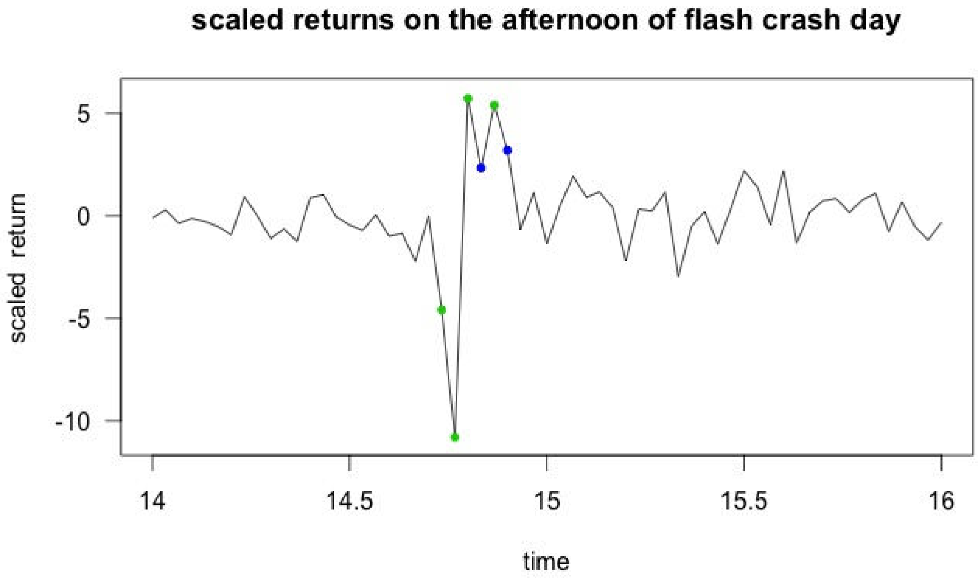



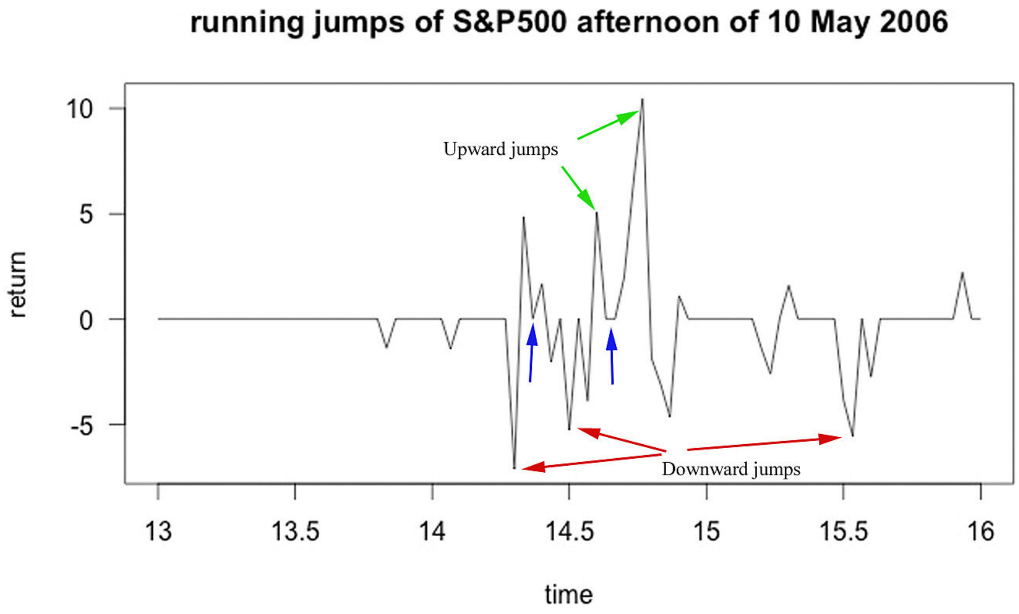

Classic Volatility Models failed to capture the mini-flash crash occurred on 10/05/2010. E.g. Dow dropped 998.5 points (9%) within minutes.

On 21/04/2015, the U.S. Department of Justice laid "22 criminal counts, including fraud and market manipulation" [8] against Navinder Singh Sarao, a trader, for significantly spoofing the algorithms.

http://www.ft.com/cms/s/0/efb897e6-40f7-11e5-9abe-5b335da3a90e.html#axzz3ogDBjsju

# Mini-Flash Crash




On this flash-crash day, the classic volatility models detected no jumps. When we applied the HP idea, we found there was one big downward jump followed by a big upward jump, which may just cancel off each other. This is why the classic approach failed to detect jumps. We also found that both the downward and upward spikes are composed of a series price movement (which we call running jumps or clustering of jumps). With HP forecasting, maybe traders could just sit back to wait for the market to bounce back instead of running around being mad.
www.youtube.com/watch?v=E1xqSZy9_41

**Q & A**

**Thank you for your attention!**